Title no. 107-S15

Comparison of One- and Two-Way Slab Minimum Thickness Provisions in Building Codes and Standards

by Young Hak Lee and Andrew Scanlon

Minimum thickness provisions for one- and two-way slabs provide a well-established approach for deflection control. Various national design codes and specifications have approached these provisions from different perspectives. Concerns have been raised about the range of the validity of current ACI Code provisions. This paper compares the ACI Code provisions with several national codes and an equation proposed by the authors to incorporate the main design variables affecting deflection control. Based on the results of the comparison, a recommendation is made to adopt the proposed equation, retaining the current values as upper limits.

Keywords: deflection; minimum thickness; reinforced concrete; serviceability.

INTRODUCTION

The ACI Code (ACI Committee 318 2008) provides minimum thickness values for one- and two-way slabs, under prescribed conditions, as a function of span length, boundary conditions, and steel yield strength as a basis for deflection control. These provisions have remained essentially unchanged since 1971 and are attractive due to their simplicity. A number of authors have raised questions about the validity of the current provisions under certain design conditions (Grossman 1981; Rangan 1982; Gilbert 1985; Hwang and Chang 1996; Scanlon and Choi 1999; Scanlon et al. 2001; Bondy 2005). To address these questions, the authors proposed a unified equation that could be applied to one- and two-way slabs as well as beams (Scanlon and Lee 2006). Building codes and standards in other parts of the world also provide minimum thickness or span-depth criteria for deflection control. The objective of this paper is to compare the current ACI provisions with the authors' proposed equation and provisions currently used in other codes and standards. The provisions selected for comparison are those incorporated in the British Standard for Design of Concrete Structures (British Standards Institution 1997), Eurocode 2: Design of Concrete Structures (British Standards Institution 2004), and the Australian Standard for Concrete Structures (AS Committee BD-002 2001). Similarities and differences among the various provisions are identified and recommendations for changes to the ACI Code are presented. The scope of this paper is restricted to one- and two-way slab systems covered by provisions of the ACI Code. Flat plates with small shear caps that do not qualify as drop panels according to ACI 318-08 could be considered as flat plates with an appropriate definition of clear span using Section 13.1.2 of ACI 318-08. A separate study is being conducted to compare code provisions for deflection control of beams.

RESEARCH SIGNIFICANCE

Minimum thickness provisions in ACI 318-08 have remained unchanged since 1971. This paper presents a

review of the current provisions, including comparisons with several national codes, and provides recommendations for changes to ACI 318-08 provisions for one- and two-way nonprestressed construction.

MINIMUM THICKNESS PROVISIONS

Minimum thickness provisions are attractive as a means of deflection control due to their simplicity. In this paper, four different codes, including ACI 318-08, BS 8110-1:1997, Eurocode 2, and Australian Standard AS 3600-2001 and the proposed approach of Scanlon and Lee (2006) are compared for one- and two-way slabs. The selected codes have been used for many construction projects, not only for their homelands but also in other countries including those in Africa, Asia, and South America. ACI 318 provisions are based on member depth, whereas other codes are based on reinforcement effective depth. To allow for comparison of the various methods considered, 1 in. (25.4 mm) was added to the effective depth for cases where minimum thickness was based on effective depth. Comparisons were thus based on total member depth. A brief description of the various code provisions is provided as follows with reference to the summary provided in Table 1.

ACI Building Code (ACI 318-08)

ACI 318-08 provides minimum thickness provisions as a fraction of span length for both one- and two-way slabs, as shown in Table 1. The minimum thickness values are independent of applied load including live and dead loads, and no limits are specified on the applicable range of span lengths. Modification factors are provided for steel yield strength and lightweight concrete.

British Standard Code (BS EN 8110-1:1997)

For the minimum thickness requirements, BS EN 8110-1:1997 provides basic span-to-effective depth ratios that vary according to support conditions, including simply supported, continuous, and cantilever. In addition to the basic span-to-effective depth ratio, a modification factor is provided for tension reinforcement determined by tensile strength of reinforcements and design ultimate moment at the center of the member (for cantilever, at the support). A modification factor for compression reinforcement is also used. Additionally, BS EN 8110-1:1997 specifies modification factors for spans

ACI Structural Journal, V. 107, No. 2, March-April 2010.

MS No. S-2008-256.R1 received April 17, 2009, and reviewed under Institute publication policies. Copyright © 2010, American Concrete Institute. All rights reserved, including the making of copies unless permission is obtained from the copyright proprietors. Pertinent discussion including author's closure, if any, will be published in the January-February 2011 ACI Structural Journal if the discussion is received by September 1, 2010.

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greater than 32.8 ft (10 m) and for flat plates, as indicated in Table 1.

Eurocode 2 (BS EN 1992-1-1:2004)

Minimum thickness in the Eurocode 2 provisions is based on a reinforcement ratio depending on whether the actual reinforcement ratio is larger than or smaller than a given reference reinforcement ratio. Because the reinforcement ratio cannot be determined until the member depth is established, an initial estimate must be made of the reinforcement ratio. If the span length exceeds 23.0 ft (7 m), the depth is multiplied by 23.0/span length (7/span length) to account for an increased self-weight of the member as span length increases. For flat plates, the depth is multiplied by 27.9/span length (8.5/span length).

The minimum thickness requirement of Eurocode 2 can be applied to both beams and slabs and depends on the reinforcement ratio calculated from moment at the center of the member and compressive strength of the concrete. The difference between one- and two-way construction, however, is not well-defined in the application of the minimum thickness equations.

Australian Standard (AS 3600-2001)

The current Australian Standard, AS 3600-2001, provides a span-to-effective depth equation as listed in Table 1, taking into account the effects of cracking, long-term effects, and load conditions. The equation is similar in form to the equation proposed by Scanlon and Choi (1999) which, in turn, is based on a simplification of the form originally proposed by Rangan (1982).

Unified equation proposed by Scanlon and Lee (2006)

Scanlon and Lee (2006) proposed a generalized minimum thickness equation for one- and two-way nonprestressed construction in terms of span-depth ratios considering

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		Minimur	n thickness of nonpr	restressed one-way slabs unless deflections are calculated					
	Simply supported	One end continuous	Both ends continuous	Cantilever					
	ℓ/20	<i>l</i> /24	ℓ/28	U 10					
	Minimum thickness of two-way slabs without interior beams								
			Without drop panels						
		Exterior panels		Interior panels					
	$f_{y,\mathrm{psi}}$	Without edge beams	With edge beams	_					
	60,000	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$					
	ℓ is span length of one-way slab ℓ_n is length of clear span measured face-to-face of supports								
ACI 318-08	For slabs with beams spanning between supports on all sides								
	Siabs without drop Slabs with drop par $0.2 \le \alpha_{fm} \le 2$ $\alpha_{fm} > 2$			$\frac{0.8 + \frac{f_y}{200,000}}{0.5\beta(\alpha_{fm} - 0.2)}$ Not less than 5 in. $\frac{8 + \frac{f_y}{200,000}}{36 + 9\beta}$ Not less than 3.5 in.					
	α_{fm} is average value of α_f for all beams on edges of panel α_f is ratio of flexural stiffness of beam section to flexural stiffness of width of slab								
AS 3600-2001	$L_{ef}/d = k_3 k_4 \left[\frac{(\Delta/L_{ef})E_c}{F_{d.ef}} \right]^{1/3}$ L_{e								

Table 1—Minimum thickness provisions for slabs (cont.)

BS 8110-1	Basic span/effective for rectangular s		Modification factor for tension reinforcement		
	Support conditions	Rectangular section	$= 0.55 + \frac{(477 - f_s)}{120(0.9 + \frac{M}{hd^2})} \le 2.0 f_s = \frac{2}{3}f_y$		
	Cantilever Simply supported Continuous	7 20 26	bd^{-j} M is design ultimate moment at center of span or, for cantilever, at support; f_s is estimated design service stress in tension reinforcement; f_y is yield strength of reinforcement; d is effective width of rectangular beam; d is effective depth; For spans exceeding 10 m, Table 3.9 should be multiplied by 10/span, except for cantilevers where the design should be justified by calculation. For flat plate, span/effective depth ratio should be multiplied by 0.9.		
Eurocode 2	If $\rho > \rho_o$	$\sqrt{f_{ck}} \frac{\rho_o}{\rho} + 32 \sqrt{f_{ck}} \left(\frac{\rho_o}{\rho} - 1 \right)$ $5 \sqrt{f_{ck}} \frac{\rho_o}{\rho - \rho'} + \frac{1}{12} \sqrt{f_{ck}} \sqrt{\frac{\rho_o}{\rho - \rho'}}$	ρ is required tension reinforcement ratio, = χη ε κ γ ρ is required tension reinforcement ratio at midspan to resist moment due to design loads (at support for cantilevers);		
Scanlon and Lee's proposal	$\frac{\ell_n}{h} = \beta \left[\left(\frac{\Delta_{inc}}{\ell} \right)_{allow} \frac{2400 k_{Dl}}{\kappa k_{AR} k_{ss} (\lambda W)} \right]$ $\frac{\ell_n}{h} = \beta \left[\left(\frac{\Delta_{inc}}{\ell} \right)_{allow} \frac{1}{\kappa k_{AR} k_{ss}} \right]$	s · · · · L(www.)) · ·	2 for one end continuous, and 48 for fixed end cantilever; $k_{DP} = 1$, except $k_{DP} = 1.35$ for slab with drop panels; $k_{CC} = 1$ except $k_{CC} = 1.35$ for column supported two-way		

applied loads, long-term multipliers, effects of cracking, and target deflection-to-span limitations. The equation is based on using an effective moment of inertia, I_e , equal to one-half of the gross moment of inertia, I_g . This approximation allows the thickness to be selected without knowing the reinforcement ratio, although an initial estimate of the depth is required to compute dead load due to self-weight. The unified equation was developed from the equation suggested by Scanlon and Choi (1999) for one-way slabs based on a simplified form of the approach proposed by Rangan (1982) for one-way construction and extended to two-way construction by Gilbert (1985).

PARAMETRIC STUDY

A parametric study was performed to evaluate the effects of design variables on the minimum thickness and to compare values calculated from the various design codes and the Scanlon and Lee (2006) unified equation. The parameters considered herein include the following.

Span length—The following span lengths were used to represent ranges typically encountered in practice:

- 10, 15, 20, 25, 30, 35, and 40 ft (3.05, 4.57, 6.10, 7.62, 9.14, 10.67, and 12.19 m) for one-way slabs;
- 10, 15, 20, 25, and 30 ft (3.05, 4.57, 6.10, 7.62, and 9.14 m) for flat plates (square);

- 10, 15, and 20 ft (3.05, 4.57, and 6.10 m) for edge supported two-way slabs; and
- Live load—Live loads are based on ASCE/SEI 7 (2005) provisions for: a) office occupancy plus allowance for partitions: 70 psf (3.4 kPa); (b) for example, assembly and restaurant: 100 psf (4.87 kPa); and (c) storage (light to heavy): 200 psf (9.74 kPa).

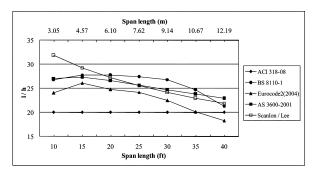
The following design parameters were held constant:

- Concrete compressive strength: 4000 psi (27.58 MPa);
- Yield strength of reinforcement: 60,000 psi (413.69 MPa);
- Superimposed dead load: 15 psf (0.73 kPa);
- Sustained live load: (a) 20 psf (0.97 kPa) for live load equal to 70 psf (3.4 kPa) and 100 psf (4.87 kPa); and (b) 50 psf (2.44 kPa) for live load equal to 200 psf (9.74 kPa); and
- Long-time multiplier λ : 2.

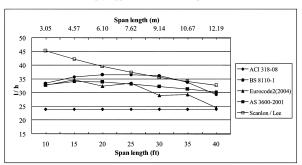
Target allowable deflections for AS 3600-2001 and the unified equation were taken as $\ell/240$ and $\ell/480$. All of the provisions considered except ACI 318 consider variation of live load in establishing minimum thickness. Design parameters considered in the various codes are summarized in Table 2. Results of the parametric study are presented in the following for one-way slabs, flat plates, and edge-supported slabs. Deflection limits are those corresponding to deflections occurring after installation of nonstructural elements.

Table 2—Design parameters considered in minimum thickness provisions

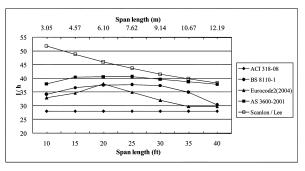
	Design parameters considered						
Design parameters	ACI 318-08	BS 8110-1	Eurocode 2	AS 3600-2001	Scanlon and Lee's proposal		
Boundary condition	Yes	Yes	Yes	Yes	Yes		
Span length	Yes	Yes	Yes	Yes	Yes		
Live load	No	Yes	Yes	Yes	Yes		
Superimposed dead load	No	No	No	No	Yes		
Allowable deflection limit	No	No	No	Yes	Yes		



(a) Simply supported: Live load = 70 psf (3.4 kPa).



(b) One end continuous: Live load = 70 psf (3.4 kPa).

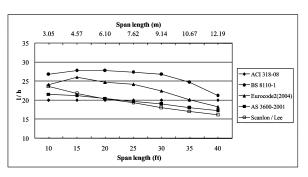


(c) Both ends continuous: Live load = 70 psf (3.4 kPa).

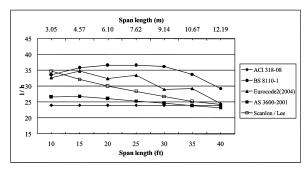
Fig. 1—Span-depth ratio as function of span length—one-way slabs, $\ell/240$: (a) simply supported: live load = 70 psf (3.4 kPa); (b) one end continuous: live load = 70 psf (3.4 kPa); and (c) both ends continuous: live load = 70 psf (3.4 kPa).

One-way slabs

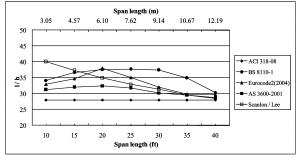
Figures 1(a) to (c) show span-depth ratios versus span length for one-way slabs with various end conditions for a constant live load of 70 psf (3.4 kPa) and a deflection limit of $\ell/240$. For this case, ACI 318 values are consistently lower than all others for spans up to approximately 40 ft (12.19 m). All provisions, except ACI 318, show a general trend of decreasing span-depth ratio with increasing span length. Figures 2(a) to (c) show the corresponding results for a deflection limit of $\ell/480$. It should be noted that the minimum



(a) Simply supported: Live load = 70 psf (3.4 kPa).



(b) One end continuous: Live load = 70 psf (3.4 kPa).



(c) Both ends continuous: Live load = 70 psf (3.4 kPa).

Fig. 2—Span-depth ratio as function of span length—one-way slabs, ℓ /480: (a) simply supported: live load = 70 psf (3.4 kPa); (b) one end continuous: live load = 70 psf (3.4 kPa); and (c) both ends continuous: live load = 70 psf (3.4 kPa).

thickness values given in ACI 318 are intended for use with slabs not supporting or attached to nonstructural elements likely to be damaged by large deflections, that is, the $\ell/240$ limit. These results suggest, however, that ACI 318 values should be satisfactory in most cases to satisfy the $\ell/480$ limit for live load up to 70 psf (3.4 kPa), except for the simply supported case where the ACI 318 values are conservative compared with other provisions up to a span of approximately 20 ft (6.10 m).

Effects of varying live load according to the provisions considered are shown in Fig. 3 for a deflection limit of $\ell/480$.

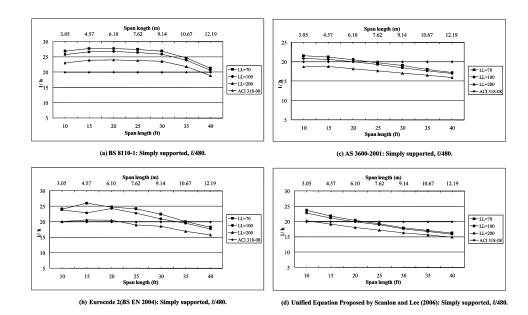


Fig. 3—Span-depth ratio as function of span length and variable live loads—one-way slabs: (a) BS 8110-1: simply supported, \$\mathcal{U}480\$; (b) Eurocode 2: simply supported, \$\mathcal{U}480\$; (c) AS 3600-2001: simply supported, \$\mathcal{U}480\$; and (d) unified equation proposed by Scanlon and Lee (2006): simply supported, \$\mathcal{U}480\$.

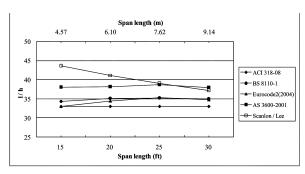
Values obtained using ACI 318, Table 9.5(a), are used as a reference, although strictly speaking, the ACI 318 values are not applicable to the deflection limit of $\ell/480$. The simply supported case is selected as the basis for comparison. In all cases except ACI 318, the calculated span-depth ratio decreases as live load increases. The BS 8110-1:1997 spandepth ratios are less conservative than ACI 318 up to a span of approximately 40 ft (12.19 m). Eurocode 2 provides spandepth ratios that are higher than the ACI 318 values for spans greater than 35 ft (10.67 m) and live load less than 100 psf (4.87 kPa). For the 200 psf (9.74 kPa) live load case, Eurocode 2 provides lower span-depth ratio than ACI 318 for spans greater than approximately 22 ft (6.71 m). AS 3600-2001 and Scanlon and Lee (2006) show similar trends providing span-depth ratios that are more conservative than the ACI 318 values over a wider range of span lengths than BS 8110-1:1997 and Eurocode 2.

Flat plates

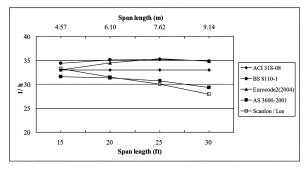
Figure 4 shows the span-depth ratio plotted against span length for an interior panel of a flat plate for $\ell/240$ and $\ell/480$ deflection limits, and a live load of 70 psf (3.4 kPa). Figure 4(a) shows that the ACI Code values are conservative compared to the other provisions for the $\ell/240$ deflection limit. For the $\ell/480$ case, however, the AS 3600-2001 and Scanlon and Lee (2006) values are more conservative for spans greater than approximately 15 ft (4.57 m). Figure 5 shows the effect of increasing live load for the $\ell/480$ limit. In all cases for heavy live loads (200 psf [9.74 kPa]), the ACI values are unconservative compared with the other provisions.

Edge-supported two-way slabs

Figures 6 and 7 show the comparison between ACI 318 and the other provisions for two-way edge-supported slabs of varying aspect ratios and live loads. In general, the ACI 318 values are seen to be conservative compared to the other provisions while there is a wide variation in the values obtained with the various provisions.



(a) Live load = $70 \text{ psf } (3.4 \text{ kPa}), \ell/240.$



(b) Live load = 70 psf (3.4 kPa), $\ell/480$.

Fig. 4—Span-depth ratio as function of span length—flat plates: (a) live load = 70 psf (3.4 kPa), V240; and (b) live load = 70 psf (3.4 kPa), V480.

DISCUSSION OF RESULTS

Results of the parametric study indicate that ACI minimum thickness values for one-way slabs and edge-supported two-way slabs are generally conservative compared with the other provisions considered for span lengths up to approximately 40 ft (12.19 m) for both the \$\mathcal{U}240 and \$\mathcal{U}480 deflection limits. It should be noted that, strictly speaking, the ACI 318-08 minimum thickness values for one-way slabs should only be used for slabs "not supporting

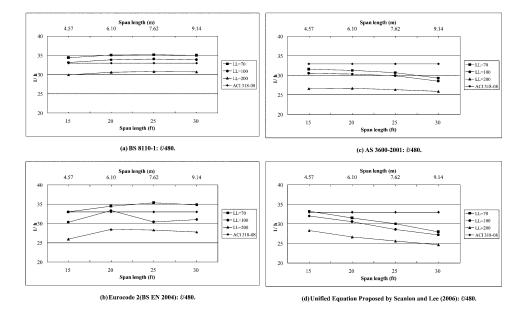
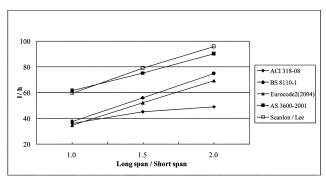
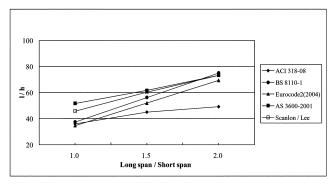


Fig. 5—Span-depth ratio as function of span length and variable live loads—flat plates: (a) BS 8110-1, \$\mu480\$; (b) Eurocode 2, \$\mu480\$; (c) AS 3600-2001, \$\mu480\$; and (d) unified equation proposed by Scanlon and Lee (2006), \$\mu480\$.



(a) Live load = 70 psf (3.4 kPa), $\ell/240$.



(b) Live load = 70 psf (3.4 kPa), $\ell/480$.

Fig. 6—Span-depth ratio as function of span length: edge-supported slabs (short span = 15 ft [4.57 m] and $\alpha_{\rm fm}$ = 2 for ACI 318-08 requirement): (a) live load = 70 psf (3.4 kPa), ℓ /240; and (b) live load = 70 psf (3.4 kPa), ℓ /480.

or attached to non-structural elements likely to be damaged by large deflections." This is consistent with the generally acceptable performance over the years of one-way slabs and edge-supported two-way slabs. For heavy live loads and heavy superimposed dead loads (greater than 100 psf [4.87 kPa]), a more detailed deflection evaluation is recommended.

The situation with respect to flat plates is somewhat different. For the $\ell/240$ limit, the ACI minimum thickness values appear to be adequate for the span range considered and a specified live load of 70 psf (3.4 kPa). For the $\ell/480$ case, however, the AS 3600-2001 and Scanlon and Lee (2006) provisions suggest that the current ACI 318 values are generally unconservative for the span and live load range considered. This is particularly the case for longer spans and higher load levels. These results suggest that the current ACI 318-08 minimum thickness values need to be reevaluated in terms of their applicability to slabs "supporting non-structural elements likely to be damaged by large deflections."

CONCLUSIONS AND RECOMMENDATIONS

Various design provisions including ACI 318-08, BS 8110-1:1997, Eurocode 2, and AS 3600-2001 and the unified equation proposed by Scanlon and Lee (2006) are compared in terms of minimum thickness for one- and two-way slabs. The effects of design parameters such as support condition, span length, and applied load are evaluated. The results indicate that ACI 318 provisions need to be revised to cover the range of design parameters that are prevalent in current practice. The results of the parametric study suggest that while these minimum thickness values are easy to apply, limitations need to be placed on the applicability of current ACI values. In particular, the ACI values for flat plates (and flat slabs) seem to be adequate for the \(\ell/240\) limit for typical spans and loading but may be inadequate in many cases to satisfy the ℓ/480 limit. It is recommended that the Scanlon/Lee equation for minimum thickness of one-way slabs, flat plates, and flat slabs be adopted by ACI 318 but not less than values given by the current limits. The advantage of the proposed equation is that it is relatively easy to apply and covers a wider range of design conditions than seems to have been anticipated when the current provision were introduced. Given the approximations involved in the proposed equation and many years of experience with the current provisions, however, it is considered prudent to retain the current minimum thickness values when the proposed equation produces a lower thickness value than the current

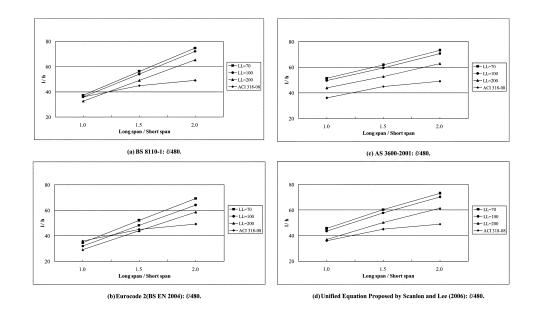


Fig. 7—Span-depth ratio as function of span length and variable live loads: edge-supported slabs (short span = 15 ft [4.57 m] and $\alpha_{\rm fm}$ = 2 for ACI 318-08 requirement): (a) BS 8110-1, V480; (b) Eurocode 2 (BS EN 2004), V480; (c) AS 3600-2001, V480; and (d) unified equation proposed by Scanlon and Lee (2006), V480.

provisions. The proposed minimum thickness equation should not be applied when slabs are over-loaded before the specified 28-day concrete strength has been reached unless appropriate adjustments to the equation have been made to account for such early age loading.

ACKNOWLEDGMENTS

This work was supported by a grant from the Kyung Hee University in 2009 (KHU-20090758).

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